Normal forms for singular dynamics

Yvain Bruned[1]

Universite de Lorraine, CNRS, IECL, F-54000 Nancy, France
Email: yvain.bruned@univ-lorraine.fr

Abstract

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Tuesday 3 June 2025

Talks take place at IECL, Salle de Conférences.

- 14h00-14h50: Tadahiro Oh.

  Normal form method for dispersive PDEs

  In this talk, I present a general overview (certainly not comprehensive) of normal form approaches in the study of nonlinear dispersive PDEs. I will first go over basic ideas for the Birkhoff normal form reductions and the Poincaré (and Poincaré-Dulac) normal form reductions and discuss some examples. Then, I will present several extensions of the Poincaré-Dulac normal form reductions:

  (i) a normal form approach to the stochastic dispersive PDEs, proving unconditional uniqueness,
  (ii) a slightly modified version of the Poincaré-Dulac normal form reductions with several applications such as (ii.a) unconditional uniqueness of solutions to modulated dispersive PDEs (without assuming any temporal regularity), improving the sewing lemma approach, and (ii.b) a normal form integrator for time-discretization of a (modulated or unmodulated) dispersive PDE.

- 14h50-15h40: Jacky Cresson.

  T.B.A.

- 15h40-16h10: Coffee break.
- 16h10-17h00: Dario Bambusi.

Almost global existence for Hamiltonian PDEs on boundaryless compact manifolds

Taking the nonlinear Klein Gordon equation as a model problem, I will present a result on the qualitative behaviour of solutions of Hamiltonian PDEs on compact boundaryless Riemannian manifolds. Precisely, one has that solutions corresponding to smooth and small initial data remain small and smooth for times of order ϵ^{-r}, $\forall r$. Here ϵ is the size of the initial datum. The proofs is based on variants of Birkhoff normal form.
I will start by reviewing the classical method of Birkhoff normal form for finite dimensional Hamiltonian systems, then I will recall the theory for equations in one space dimensions and finally I will present the ideas leading to the theory in higher space dimension.
Joint work with J. Bernier, B. Grebert, R. Imekratz.

- 19h30-22h30: Conference dinner at Grand Café Foy.

Wednesday 4 June 2025

Talks take place at IECL, Salle de Conférences.

- 9h30-10h20: Katharina Schratz.

  Resonances as a computational tool

  A large toolbox of numerical schemes for dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene such as for problems at low regularity and high oscillations. Classical schemes fail to capture the oscillatory nature of the solution, and this may lead to severe instabilities and loss of convergence. In this talk I present a new class of resonance based schemes. The key idea in the construction of the new schemes is to tackle and deeply embed the underlying nonlinear structure of resonances into the numerical discretization. As in the continuous case, these terms are central to structure preservation and offer the new schemes strong geometric properties at low regularity.

- 10h20-10h50: Coffee break.

- 10h50-11h40: Tristan Robert.

  T.B.A

- 11h40-12h30: Frédéric Fauvet.

Explicit linearization of multi-dimensional germs and vector fields through Ecalle's tree expansions

We provide explicit formulas of non-recursive type for the linearizing transformations of a non-resonant analytic germ of diffeomorphism at a fixed point or a non-resonant analytic germ of vector field at a singular point, in any complex dimension. The formal expressions we obtain rely on a part of Ecalle's tree-based combinatorics called "arbomould calculus" and they have the same shape for dynamical systems with discrete or continuous time respectively. They allow us to recover in a straightforward way, under Bruno's arithmetical condition, the sharpest estimates so far for the domains of convergence of the analytic linearizing changes of variables.
Joint work with Frédéric Menous, U. Paris–Saclay and David Sauzin, CNRS–Paris Observatory.

- 12h30-14h30: Lunch.

  Talks take place at IECL, Salle Döblin.

- 14h30-15h20: Yvain Bruned.

  Decorated trees and arborification for dispersive PDEs normal forms

  In this talk, we will introduce a decorated tree formalism for describing the Duhamel iterates of a dispersive PDEs in Fourier space. Then, we will present an arborification map which transforms the trees into linear combination of words. This allows us to provide a derivation of Poincaré-Dulac normal form. Such a formalism has also been used for computing cancellations in Wave turbulence.

- 15h20-15h50: Coffee break.

- 15h50-16h40: Anne-Sophie De Suzzoni.

  Trivial resonances for a system of Klein-Gordon equations and statistical applications

  In the derivation of the wave kinetic equation coming from the Schrödinger equation, a key feature is the invariance of the Schrödinger equation under the action of $U(1)$. This allows quasi-resonances of the equation to drive the effective dynamics of the statistical evolution of solutions to the Schrödinger equation. In this talk, I will give an example of an equation that does not have the same invariance as the Schrödinger equation, and I will show that in this example, exact resonances (always) take precedence over quasi-resonances, so that the effective dynamics of the statistical evolution of the solutions are not kinetic. However, these dynamics are not linear (let alone trivial). I will present the problem and the ideas involved in deriving the effective dynamics and some elements of proof: in particular, I will describe the representation of solutions of the initial equation in diagrammatic form. This talk is based on a joint work

with Annalaura Stingo (X) and Arthur Touati (Bordeaux).

- 16h40-17h30: Kurusch Ebrahimi-Fard.

  New perspectives on the Magnus expansion

  Originally proposed by Wilhelm Magnus in 1954 to express the logarithm of solutions to linear operator-valued initial value problems, the Magnus expansion has since become a fundamental tool across physics, chemistry, and engineering. In recent decades, its mathematical structure has been unfolded, revealing deep ties to algebraic, combinatorial, and geometric frameworks. In particular, modern formulations based on Hopf, pre- and post-Lie algebra structures not only unify various computational approaches but also allow for a conceptual reinterpretation via crossed morphisms, with further links to Gavrilov's K-map.

Thursday 5 June 2025

Talks take place at IECL, Salle de Conférences.

- 9h00-9h50: Loïc Foissy.

  Strange pre- and post-Lie structures on rooted trees

  We present a construction of pre-Lie on rooted trees whose edges and vertices are decorated, with a grafting product twisted by an action of a map acting on both edges and vertices. We show that this construction indeed gives a pre-Lie algebra if, and only if, a certain commutation relation is satisfied. Then, this pre-Lie algebra can be extended as a post-Lie algebra through a semi-direct product.
  A particular example is used for normal forms in the study of stochastics PDEs. Here, the set of decorations of edges and vertices is $\mathbb{N}^{d+1}$ and the acting map is the exponentiation of a simpler map.

- 9h50-10h20: Coffee break.

- 10h20-11h10: Jacob Armstrong-Goodall.

  An Explicit Formula for the Birkhoff Normal Form

  Many nonlinear dispersive equations can be studied as Hamiltonian systems with kinetic and 'interaction' terms. When symplecticity is assured, the Birkhoff normal form reduction is a classical approach for studying the long term dynamics of solutions to these systems. The normal form is derived via a canonical transformation which removes non-resonant frequency interactions up to an arbitrary order. By encoding such a transform combinotorially I will introduce an explicit formula for the Birkhoff normal form in terms of decorated trees. The cubic Schrödinger equation will be presented as a concrete example of a Hamiltonain system for which this approach is useful.

- 11h10-12h00: Leonardo Tolomeo.

T.B.A.